Homework 4:

**Activity 12**:

1. Write matlab code to solve the steady-state heat equation (convection to the outside laterally) with Neumann BCs using the finite difference method. Submit your code and the graph of the solution. Label the axes.

2. An insulated heated rod with a uniform heat source can be modeled by Poisson’s equation:

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Given a heat source *f*(*x*) = 25 and boundary conditions *u*(0) = 40 and *u*(10) = 200.

1. Write the finite difference system of equations with *h* = 2.
2. Solve using matlab. Submit a graph of the solution. Label the axes.
3. Solve in matlab using *h* = 0.2 (or larger if required by matlab) Submit your code and graph of the solution. Label the axes

3. The following is a simple reaction-diffusion equation describing the steady-state concentration, *c*, of a substance that reacts in a long reactor and disperses axially:

where *D* = 1.5 = the dispersion coefficient,

*k* = 5 = reaction rate

*L* = 100.

Boundary conditions are given by *c*(0) = 0.1 and *c*(*L*) = 1.

1. Write the system of equations.
2. Solve in matlab using finite differences with *h* = 20.
3. Repeat with *h* = 2 (or as small as matlab will permit). Submit code and graph of the solution.

**Partial Differential Equations**:

4. Problem from class on 11/27 with heat flow proportional to the temperature difference at *x* = 1. Submit code and a graph of the solution with labeled axes. Use forward difference, finite difference method.

5. Consider the equation

1. Show that is the solution of the problem, i.e. satisfies the PDE and the side conditions.
2. Show the modification of the forward difference, finite difference method to the discretization for this problem.
3. Solve the problem using forward difference, finite difference method in matlab with dx = 0.1 and dt = 0.01 Submit your matlab code and a graph of the solution.
4. For what step sizes dt, is the forward difference method stable, given dx = 0.1? Check this out in your matlab code by changing dt and seeing what happens. Write down what you observe.

6. Now consider the problem in 5. above again.

i) Use the backward difference method to the solve this problem. You may use the code posted and discussed in class on 11/29. Submit your graph only.

ii) Using the backward difference method, make a table of the exact value, the approximate value, and the error at *x* = 0.3, *t* = 1 for step sizes dx = 0.1 and dt = 0.02, 0.01, 0.005.

7. Consider the equation , insulated on both ends.

1. Show the modification to the discretization for this problem.
2. Solve the problem using forward differences in matlab with dx = 0.1 and dt = 0.01 Submit your matlab code and a graph of the solution.
3. For what step sizes dt, is the forward difference method stable, given dx = 0.1? Check this out in your matlab code by changing dt and seeing what happens. Write down what you observe.

8. Now consider the problem in 7. above again.

i) Use the backward difference method to the solve this problem. You may use the code posted and discussed in class on 11/29. Submit your graph only.

ii) Using the backward difference method, make a table of the exact value, the approximate value, and the error at *x* = 0.3, *t* = 1 for step sizes dx = 0.1 and dt = 0.02, 0.01, 0.005.